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Lesson 13: Interpreting Graphs of Functions

Student Outcomes

* Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals.

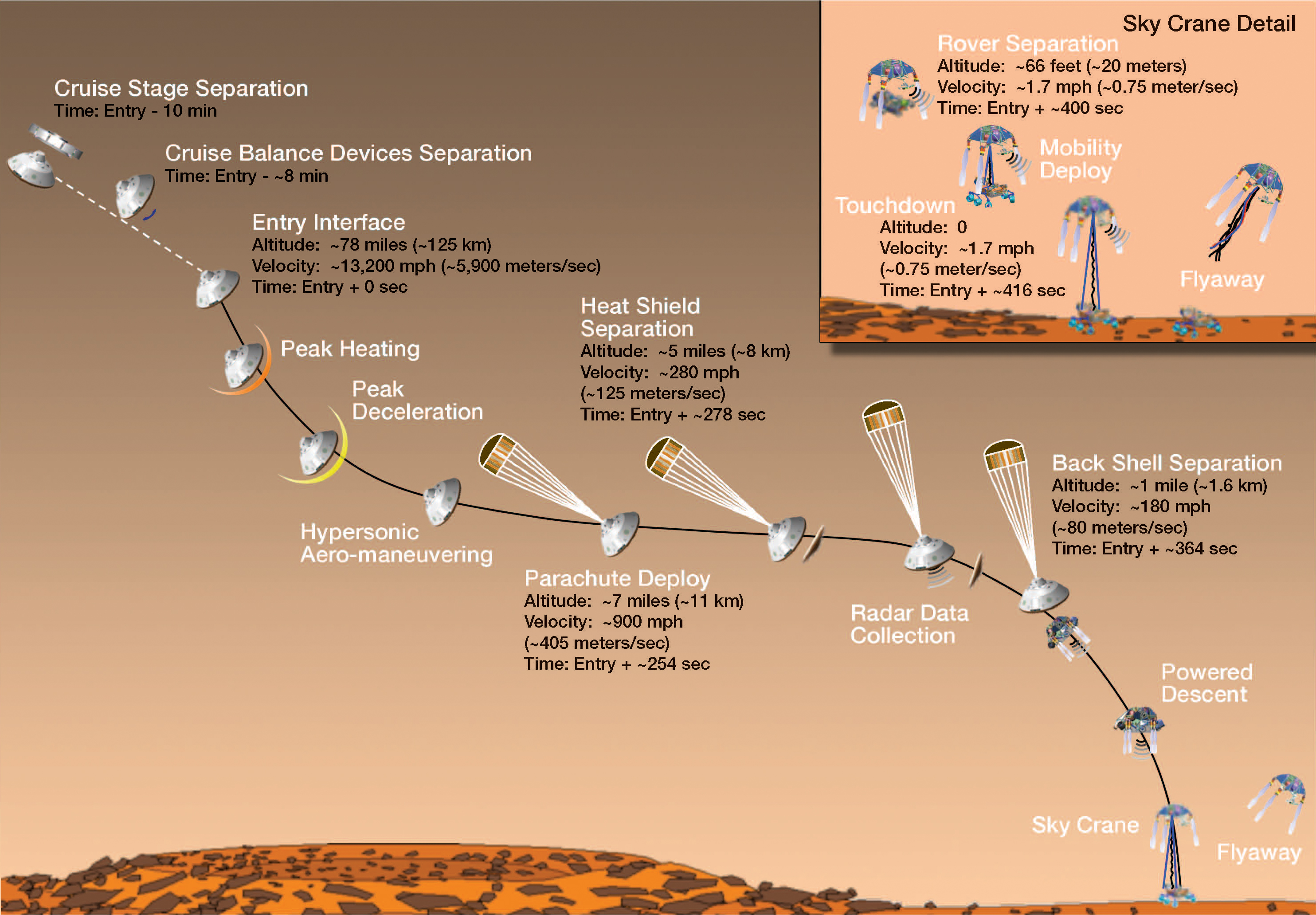
Lesson Notes

This lesson uses a graphic created to show the general public the landing sequence for the Mars Curiosity Rover which landed successfully on Mars in August 2012. For more information, visit <http://mars.jpl.nasa.gov/msl/>. For an article related to the graphic in this lesson see <http://www.nasa.gov/mission_pages/msl/multimedia/gallery/pia13282.html>. Here is an animation that details Curiosity’s descent: <http://mars.jpl.nasa.gov/msl/mission/timeline/edl/>. The first three minutes of this video show a simulation of the landing sequence <http://www.jpl.nasa.gov/video/index.php?id=1001>. Students are presented with a problem: Does this graphic really represent the path of the Curiosity Rover as it landed on Mars? And, how can we estimate the altitude and velocity of the Curiosity Rover at any time in the landing sequence? To formulate their model, students will have to create either numeric or graphical representations of height and velocity. They will need to consider the quantities and make sense of the data shared in this graphic. During this lesson, you will be presenting them with vocabulary to help them interpret the graphs they create. To further create context for this lesson, you can share this article from *Wired* <http://www.wired.com/thisdayintech/2010/11/1110mars-climate-observer-report/> with your students. It explains an earlier mishap by NASA that cost billions of dollars and two lost explorers that was due to a measurement conversion error. Scientists carefully model all aspects of space travel using mathematical functions, but if they do not attend to precision, it can lead to big mistakes.

Classwork

Classwork

This graphic was shared by NASA prior to the Mars Curiosity Rover landing on August 6, 2012. It depicts the landing sequence for the rover’s descent to the surface of the planet.



If students are having a difficult time reading the information on this graphic, go to the website and share the link or project the image in your classroom. (There is also a printer-friendly graphic at the end of this lesson.)

Discussion (5 minutes)

**PROBLEM:** Read through the problem as a whole class, and have students begin to discuss how they will create a model. Suggested discussion questions to further clarify their work are listed below.

* What information is available to you in this graphic?
  + *The graphic contains the altitude and velocity at various times. Various landing stages are named. Time 0 is at the point called Entry Interface.*
* What information is in the box in the upper right corner?
  + *This box contains detailed information about the final seconds of the landing sequence.*
* Why are there negative time values? Should other quantities be measured with negative numbers?
  + *The creators of this graphic are referencing time since entry interface began. The time associated with stages before this stage would be negative. Velocities shown are all positive, but a calculation of average velocity using the altitude as the distance function shows that the velocities listed in the graphic really should be negative. Direct your students to use negative values for velocities.*
* What does this symbol mean?
  + *This symbol means approximately.*
* Which units, metric or customary, will make this problem easier to understand?
  + *Depends: 13,200 mph is easier to comprehend when the Curiosity Rover is moving fast, but 0.75 meters/sec is easier to comprehend when the Curiosity Rover is just about to land. In general, it is easier to be more accurate with metric units; they also work naturally with the decimal system (since the metric system is based upon powers of 10). For reasons like these, metric units are the preferred system of measurements in science, industry, and engineering. You might suggest to your students that metric units may be easier to graph since time is measured in seconds and the velocity is measured in meters per second. If you use customary units, then miles per hour would have to be converted to feet per second or miles per second. Regardless of the choice, scaling this graph will make for interesting choices that students need to attend to.*

Does this graphic really represent the landing path of the Curiosity Rover? Create a model that can be used to predict the altitude and velocity of the Curiosity Rover 5, 4, 3, 2, and 1 minute before landing.

**Exploratory Challenge (20 mintues)**

**FORMULATE AND COMPUTE:** During this phase of the lesson, students should work in small groups. Focus the groups on creating a tabular and graphical representation of the altitude and velocity as functions of time since entry interface began. You may have them work on large pieces of chart paper, use appropriate technology (e.g., graphing calculator or computer spreadsheet software), or use another technique of your choice to create the tables and graphs. Have each group present their findings to the rest of the class. The discussion will need to focus on the choices your students make as they construct the tables and graphs. You may want to present the option at some point to create a graph where the velocity is negative. The sample solution provided below assumes the velocity to be negative.

Exploratory Challenge

Create a model to help you answer the problem and estimate the altitude and velocity at various times during the landing sequence.

As groups present their work, discuss the following questions.

* How did you decide on your units? How did you decide on a scale?
  + *Answers will vary.*
* Would it make sense to connect the points on the graphs? Why?
  + *It would make sense because you could measure the velocity and altitude at any point in time during the landing sequence.*
* How would you describe the velocity graph? How would you describe the altitude graph?
  + *The velocity graph appears below the -axis, and it gets closer to the -axis as time passes. The altitude gets closer to the t-axis as time passes.*

Give groups time to refine their models after seeing how other groups solved this problem. Regardless of the presentation medium, make sure students are presenting accurate graphs and tables with variables named, axes scaled, and graphs labeled and titled. Sample graphs and tables are shown below. Additional samples are provided in the Exercises section.

|  |  |  |
| --- | --- | --- |
| Mars Curiosity Rover Landing Sequence | | |
| Time (s) | Altitude (m) | Velocity (m/s) |
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At this point we do not expect students to use the vocabulary of increasing or decreasing to describe the graphs. We will save this for the discussion.

Discussion (5 minutes)

Select one set of the student graphs. Annotate the graphs to show the intervals where the functions are increasing and decreasing, the intervals where the function’s values are positive and negative, and the t- and y-intercepts. There will only be intervals where the velocity function is increasing and negative if you create a graph with negative velocity. Remind students of the definitions of increasing and decreasing functions, positive and negative shown below. A sample solution is shown after the definitions.

(Note: If you choose, you may also use this discussion to introduce interval notation. This can be done by explaining the meanings of and as exclusive and inclusive, and then asking students to sketch example intervals on a number line, such as ; ; . The intervals in this lesson may also be named by students either in words or using set-builder notation, if that is preferred.)

Let be a function whose domain and range are the subsets of the real numbers.

* A function is called *increasing* on an interval if whenever in .
* A function is called *decreasing* on an interval if whenever in .
* A function is called *positive* on an interval if for all in .
* A function is called *negative* on an interval if for all in .

Decreasing for all

Positive for all

Negative for all

-intercept

-intercept

Constant for on

Exercises 1–6 (10 minutes)

Remind students of the original problem questions, and have them compute their results and explain how they got the answer. To generate the table in Exercise 2, students may need to produce a second graph of the last three or four data points. This is fairly easy to do if students are using technology to create their graphs. Alternatively, students could interpolate values from the tables as well. Regardless of the approach, students should be attending to precision. Work with groups to really think about and determine a good method for getting a decent estimate.

Exercises

1. Does this graphic really represent the landing path of the Curiosity Rover?

No. The height is not scaled appropriately in this graphic. According to the video it also looks as if the Curiosity Rover rises a bit when the parachute is released and when the sky crane engages.

1. Estimate the altitude and velocity of the Curiosity Rover , , , , and minute before landing. Explain how you arrived at your estimate.

We used the graph and rounded the landing time to be at minutes after entry interface. The table shows the altitude and velocity.

|  |  |  |
| --- | --- | --- |
| Time since entry interface | Altitude  (miles) | Velocity  (mph) |
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|  |  |  |
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**INTERPRET AND VALIDATE:** To help students interpret and validate or refute their work, show one of the videos or animations listed in the lesson notes. Have them reconsider any of their solutions based on this new information, and give them time to make any revisions they deem necessary.

1. Based on watching the video/animation, do you think you need to revise any of your work? Explain why or why not, and then make any needed changes.

Answers will vary. Some students might suggest that the period of rapid descent and deceleration cannot be linear. Some may suggest that their graphs take into account the period of upward motion when the parachute is released and the sky crane engages.

1. Why is the graph of the altitude function decreasing and the graph of the velocity function increasing on its domain?

The altitude values are getting smaller as the time values are increasing. The velocity values are getting larger as the time values are increasing.

1. Why is the graph of the velocity function negative? Why does this graph not have an -intercept?

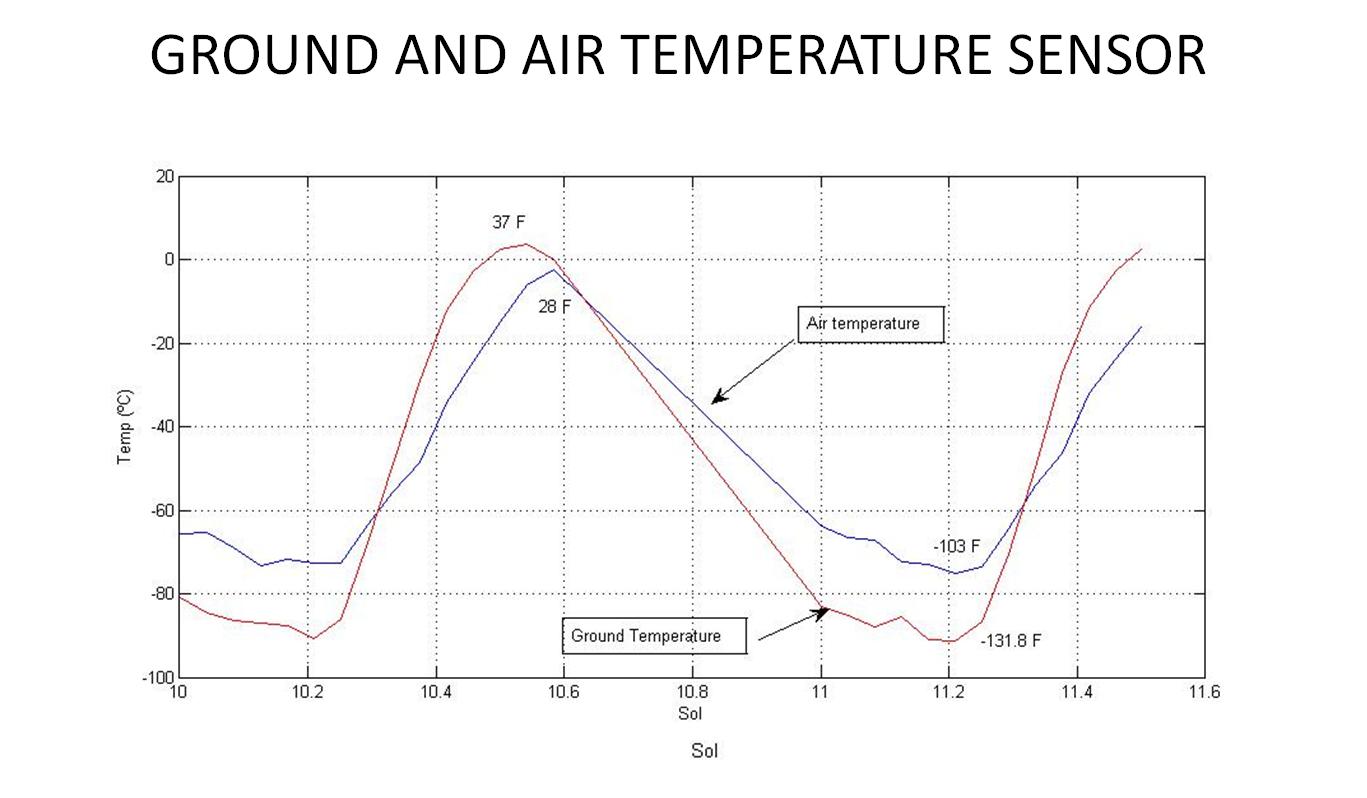
The graph is negative because we represent velocity as a negative quantity when the distance between two objects (in this case the Curiosity Rover and the surface of Mars) is decreasing. This graph does not have an -intercept because it is traveling at mph when it touches the surface of the planet.

1. What is the meaning of the -intercept of the altitude graph? The -intercept?

The -intercept is the time when the Curiosity Rover lands on the surface of Mars. The -intercept is the height of the Curiosity Rover when entry interface begins.

Exercises 7–12

Use these exercises as time permits. They will allow students to practice identifying the key features of graphs. They involve the temperature data collected on the surface of Mars. A sol is a Martian day. The length of a sol varies as it does on Earth with the mean time of sol being hours minutes and seconds.

[](http://www.google.com/url?sa=i&rct=j&q=mars+curiosity+graphs&source=images&cd=&cad=rja&docid=JG3KTa2C-7C5EM&tbnid=BszWVkmpV87P_M:&ved=0CAUQjRw&url=http://commons.wikimedia.org/wiki/File:PIA16081-Mars_Temp_Graph-Curiosity_Rover-REMS-20120821.jpg&ei=4OICUuHSDuOVjALO5YH4Dw&bvm=bv.50500085,d.cGE&psig=AFQjCNF-2er6JIsmTX0mMchu3ltuIYAW7w&ust=1376007262439774)A Mars rover collected the following temperature data over Martian days. A Martian day is called a sol. Use the graph to answer the following questions.

1. Approximately when does each graph change from increasing to decreasing? From decreasing to increasing?

Increasing to decreasing: Air: approximately and , Sol. Ground: Approximately and Sol.

Decreasing to increasing: Air: approximately Sol, Sol, and Sol.

1. When is the air temperature increasing?

Air temperature is increasing on the interval , , and .

1. When is the ground temperature decreasing?

Ground temperature is decreasing on the interval, , and .

1. What is the air temperature change on this time interval?

The high is and the low is . That is a change of . Students might also answer in Celsius units.

1. Why do you think the ground temperature changed more than the air temperature? Is that typical on Earth?

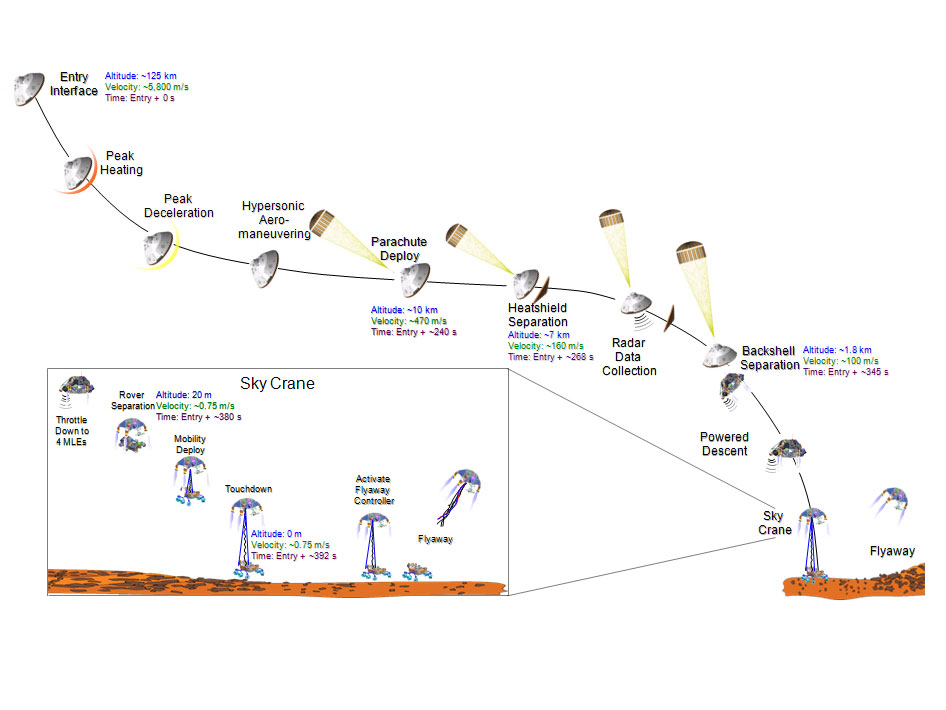
Student responses will vary.

1. Is there a time when the air and ground were the same temperature? Explain how you know?

The air and ground temperature are the same at the following times: Sol, Sol, and Sol.

Exit Ticket (5 minutes)

Teachers: Please use this graphic if the other colored graphic does not display properly.



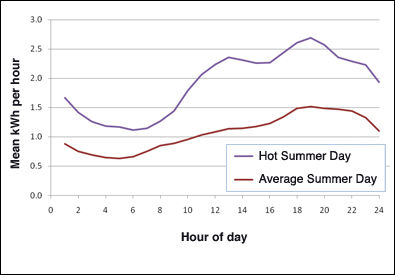
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Lesson 13: Interpreting Graphs of Functions

Exit Ticket

1. Estimate the time intervals when mean energy use is decreasing on an average summer day. Why would power usage be decreasing during those time intervals?

Power Usage on a Typical Summer Day in Ontario, Canada

[](http://www.google.com/url?sa=i&rct=j&q=energy+use+graph&source=images&cd=&cad=rja&docid=I63X9Ib95E6X6M&tbnid=lvU-h-kZ7VmtMM:&ved=0CAUQjRw&url=http://www.nrc-cnrc.gc.ca/eng/dimensions/issue6/smart_technology.html&ei=27DqUdG1FouCyAGdkYGYAg&bvm=bv.49478099,d.aWc&psig=AFQjCNFAQTvDwRDSdwe6Laiz7UotuDarqg&ust=1374421535421046)

Source: National Resource Council Canada, 2011

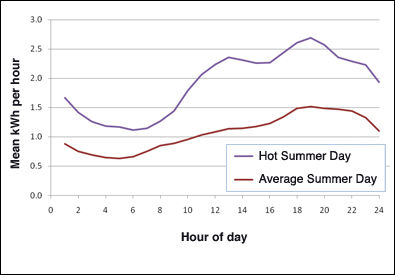
1. The hot summer day energy use changes from decreasing to increasing and from increasing to decreasing more frequently than it does on an average summer day. Why do you think this occurs?

Exit Ticket Sample Solutions

1. Estimate the time intervals when mean energy use is decreasing on an average summer day. Why would power usage be decreasing during those time intervals?

Energy use is decreasing from hour 1 to hour 5 and from hour 19 to hour 24. It is nighttime when the temperatures outside are dropping.

Power Usage on a Typical Summer Day in Ontario, Canada

[](http://www.google.com/url?sa=i&rct=j&q=energy+use+graph&source=images&cd=&cad=rja&docid=I63X9Ib95E6X6M&tbnid=lvU-h-kZ7VmtMM:&ved=0CAUQjRw&url=http://www.nrc-cnrc.gc.ca/eng/dimensions/issue6/smart_technology.html&ei=27DqUdG1FouCyAGdkYGYAg&bvm=bv.49478099,d.aWc&psig=AFQjCNFAQTvDwRDSdwe6Laiz7UotuDarqg&ust=1374421535421046)

Source: National Resource Council Canada, 2011

1. The hot summer day energy use changes from decreasing to increasing and from increasing to decreasing more frequently than it does on an average summer day. Why do you think this occurs?

Perhaps people turned down the air when they came home to conserve electricity but increased it later so they could sleep because the temperature still had not cooled down.

Problem Set Sample Solutions

The first exercise in the problem set asks students to summarize their lesson in a written report to conclude the modeling cycle.

1. Create a short written report summarizing your work on the Mars Curiosity Rover Problem. Include your answers to the original problem questions and at least one recommendation for further research on this topic or additional questions you have about the situation.

Student responses will vary.

1. Consider the sky crane descent portion of the landing sequence.
   1. Create a linear function to model the Curiosity Rover’s altitude as a function of time. (What two points did you choose to create your function? Why?)

For the function , let represent the altitude at time . .

* 1. Compare the slope of your function to the velocity. Should they be equal? Explain why or why not.

The slope of the linear function and velocity in the graphic are not equal. If we assume that the velocity is constant (which it is not), then they would be equal.

* 1. Use your linear model to determine the altitude one minute before landing. How does it compare to your earlier estimate. Explain any differences you found.

The model predicts meters. The earlier estimate was mile ( km) and was close to a given data point. The model would only be a good predictor during the sky crane phase of landing.

1. The exponential function could be used to model the altitude of the Curiosity Rover during its rapid descent. Do you think this model would be better or worse than the one your group created? Explain your reasoning.

Answers will vary depending on the class graphs. It might be better because the Curiosity Rover did not descend at a constant rate, so a curve would make more sense than a line.

1. For each graph below, identify the increasing and decreasing intervals, the positive and negative intervals, and the intercepts.
   1. Decreasing interval , positive interval , negative interval , -intercept , -intercept .



* 1. Increasing intervals , , decreasing interval , positive intervals , , negative interval , -intercept , -intercept and .

